THERMAL CONDITIONS AND THERMOMECHANICAL STRENGTH OF ACTIVE GLASS ELEMENTS OF A LASER

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Relations are derived for the temperature distribution in an active glass element of a laser operating in the single-pulse and periodic modes. The effect of rapid cooling is discussed. The limiting temperature gradient in barytic crown glass is found on the basis of the theory of elasticity.

The increased laser output power achieved through increases in the pumping energy, the efficiency of the illuminator coupling, and the optical density of the active substance results in increased heat evolution, while the finite thermal conductivity of the substance leads to a temperature gradient over the cross section of the active element. Heat evolution and transfer are not important for single-pulse lasers, but they are of primary importance for powerful periodic-mode lasers. In several studies of this topic [1-5], only the temperature distribution over the cross section of the rod has been discussed.

When a ruby active element is used, the temperature gradient increases with the number of pulses, degrading the generation parameters until the generation is completely cut off, primarily in the central part of the rod. The potentialities of ruby can be improved by cooling it with liquid nitrogen or liquid hydrogen [3]. The periodic-mode operation of a glass active element is more complicated, both with respect to the cooling and the effect of the temperature gradient, which results in thermomechanical destruction of the rod when it reaches a certain level.

Heat Distribution in the Active Element. When there is Newtonian heat transfer with the surrounding medium, and the physical properties of the active substance and the coolant are constant, the temperature distribution in the active element of the laser, which may be assumed to be an infinite cylinder, is [6]

$$T(r,\ t) = T_c + \frac{q}{c\gamma}\sum_{n=1}^{\infty} A_n J_0\left(\mu_n \frac{r}{R}\right)\exp(-\mu_n^2\,\mathrm{Fo}). \tag{1}$$

Figure 1 shows the nature of the temperature distribution and its time dependence for various cooling rates. It is important to note that both the temperature gradient and the rate of thermal relaxation increase with increasing heat-flow rate. The relative change in the temperature during cooling is

$$\frac{T(r,\ t) - T_c}{T(r,\ 0) - T_c} = \sum_{n=1}^{\infty} A_n J_0\left(\mu_n \frac{r}{R}\right)\exp(-\mu_n^2\,\mathrm{Fo}). \tag{2}$$

Thermal relaxation at the axis of the rod requires the most time. For this case, using only the first term in the series, we define the time required for the relative temperature at the axis to reach

$$m = \frac{T(0,\ t) - T_c}{T(0,\ 0) - T_c} \tag{3}$$

by

$$t = -\frac{R^2}{\mu_1^2 a}\ln m. \tag{4}$$

For a glass rod 0.5 cm in radius, the temperature at the rod axis falls to half its value in 6.5 sec, 9.5 sec,

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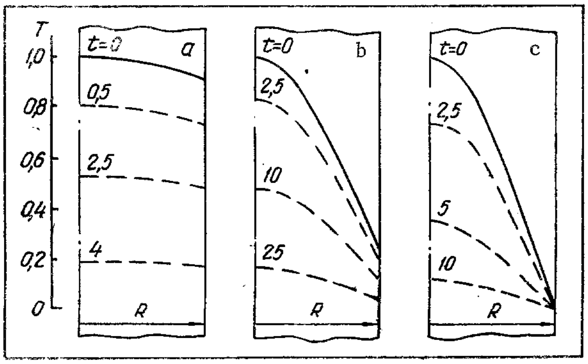

Fig. 1

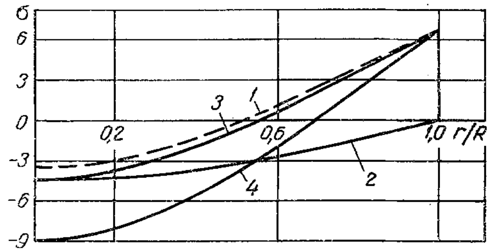

Fig. 2

Fig. 1. Radial distribution of the relative temperature of a rod for various cooling rates and time dependence of the relative temperature. a) For Bi = 0.2 (t, min); b) Bi = 5 (t, sec); c) Bi = ∞ (t, sec).

Fig. 2. Relative distribution of the thermomechanical stresses over the rod cross section. 1) Equivalent; 2) radial; 3) tangential; 4) axial.

and 1.6 min for Bi = ∞, 5, and 0.2, respectively; for R = 1.5 cm, the times are 1, 1.4, and 15 min, respectively. The temperature falls to one-tenth its value in 22 sec and 3.3 min in rods of these radii at Bi = ∞.

High-power lasers with thick rods (R ≥ 2 cm) operate in the single-pulse mode. For such operation, rapid cooling could reduce the interval required between pulses.

When a laser is operated in the periodic mode, and when the interval between pulses is less than the thermal-relaxation time, both the absolute temperature of the active element and the temperature drop between the axis and surface increase with the number of pulses until a steady state is reached. Because of the low thermal conductivity of glass and the brevity of the heat evolution in comparison with the interval between pulses, the cooling during the heat evolution can be completely neglected for practical problems at the current laser operating frequencies (10-100 Hz). In this case, the temperature distribution at the j-th pulse is

$$T_j(r,\ t) = T_c + \frac{q}{c\gamma} \sum_{n=1}^{\infty} \frac{1 - \exp\left(-j\mu_n^2 \frac{at_0}{R^2}\right)}{1 - \exp\left(-\mu_n^2 \frac{at_0}{R^2}\right)} A_n J_0\left(\mu_n \frac{r}{R}\right) \exp\left(-\mu_n^2 \frac{at}{R^2}\right). \tag{5}$$

In the steady state ($j \to \infty$), Eq. (5) becomes

$$T_{ss}(r,\ t) = T_c + \frac{q}{c\gamma} \sum_{n=1}^{\infty} \frac{\exp\left(-\mu_n^2 \frac{at}{R^2}\right)}{1 - \exp\left(-\mu_n^2 \frac{at_0}{R^2}\right)} A_n J_0\left(\mu_n \frac{r}{R}\right) \tag{6}$$

and

$$\Delta T_{ss}(t) = \frac{q}{c\gamma} \sum_{n=1}^{\infty} \frac{\exp\left(-\mu_n^2 \frac{at}{R^2}\right)}{1 - \exp\left(-\mu_n^2 \frac{at_0}{R^2}\right)} A_n [1 - J_0(\mu_n)]. \tag{7}$$

When cooling during the heat evolution is taken into account, the nature of the temperature distribution can be determined from the equations of [2] with the proper value of the coefficient A_n for the cooling employed.

The time required to reach the steady state in the range

$$m = \frac{T_{ss} - T_j}{T_{ss}} \tag{8}$$

TABLE 1. Values of the Summation in Eqs. (6) and (7) for a Pulse Repetition Frequency of 1 Hz and for Various Cooling Rates

Bi	R=0,5 cm			R=0,2 cm		
	at rod axis	at rod surface	drop	at rod axis	at rod surface	drop
0,2	143,942	130,594	13,349	23,488	21,315	2,173
1,0	41,275	27,688	13,589	7,006	4,888	2,118
5,0	19,456	5,717	13,639	3,465	1,378	2,087
10	16,757	3,123	13,634	2,971	0,956	2,015
50	14,399	0,677	13,721	2,488	0,275	2,221
100	14,132	0,347	13,786	2,428	0,144	2,284
∞	13,870	0	13,870	2,386	0	2,386

TABLE 2. Values of the Summation in Eq. (6) for r = 0 and Bi = 100 at t = 0

Frequency, Hz	Interval between sec	Σ	
		R=0,5 cm	R=0,2 cm
0,2	5	2,9	0,8
1	1	14,1	2,4
5	0,2	68,9	11,2
10	0,1	148,3	22,5
50	0,02	723,0	170,4
100	0,01	1385,6	222,0

is given by Eq. (4) when one term of the series is used, and the number of pulses is

$$j = -\frac{R^2}{\mu_1^2 a t_0} \ln m. \tag{9}$$

The steady state is reached sooner, the narrower the rods, the longer the interval between pulses, and the more rapid the cooling. After the steady state is reached, the cooling rate does not affect the temperature drop, but it does determine the absolute temperature of the active element (Tables 1 and 2). The value of the summation was calculated for six terms of the series. As is evident, the temperature along the axis of the active element increases rapidly with frequency, even when the surface is cooled rapidly. For barytic crown rods 1 and 0.4 cm in diameter ($c\gamma = 2.27$ J/cm^3·deg), an axial temperature of 1000°C will be attained at a heat evolution per unit volume per pulse of 9.8 and 1.6 J/cm^3, respectively.

Forced-water cooling is the best for use with a glass active element, offering an efficiency two orders of magnitude above forced-air cooling under similar conditions [7]. A value of Bi ≥ 100 is achieved at a flow rate above 2 m/sec. A further intensification of the cooling of the lateral surface slightly increases the heat transferred from the interior of the rod, which is limited by the thermal conductivity of the glass. The use of low-temperature coolants such as liquid hydrogen (−252.7°C) or liquid nitrogen (−195.8°C) is not advisable, since the thermal conductivity of glass is lower by a factor of 4.5 and 2, respectively, than at room temperature [8].

Thermal Stresses and Thermomechanical Strength. Substituting series (1) into the thermal-stress equations of [9], and replacing the absolute temperature at each point of the cross section by the temperature drop, we find the following stresses for a rod with free ends: a radial stress of

$$\sigma_r = \frac{\alpha' E \Delta T}{1-\nu} \sum_{n=1}^{\infty} \exp(-\mu_n^2 \mathrm{Fo}) A_n \frac{1}{\mu_n} \left[J_1(\mu_n) - \frac{R}{r} J_1\left(\mu_n \frac{r}{R}\right)\right], \tag{10}$$

a tangential stress of

$$\sigma_\theta = \frac{\alpha' E \Delta T}{1-\nu} \sum_{n=1}^{\infty} \exp(-\mu_n^2 \mathrm{Fo}) A_n \left[\frac{1}{\mu_n} J_1(\mu_n) - J_0\left(\mu_n \frac{r}{R}\right) + \frac{1}{\mu_n}\frac{R}{r} J_1\left(\mu_n \frac{r}{R}\right)\right], \tag{11}$$

and an axial stress of

$$\sigma_z = \frac{\alpha' E \Delta T}{1-\nu} \sum_{n=1}^{\infty} \exp(-\mu_n^2 \mathrm{Fo}) A_n \left[\frac{1}{\mu_n} J_1(\mu_n) - J_0\left(\mu_n \frac{r}{R}\right)\right]. \tag{12}$$

The maximum thermal stresses occur at $\exp(-\mu_n^2 \mathrm{Fo}) = 1$, i.e., when the cooling starts.

Figure 2 shows the stress distribution over the cross section of the rod. The equivalent stress is found from

$$\sigma_{eq} = \sigma_{max} - \varepsilon\sigma_{min}. \tag{13}$$

For glass, we have $\varepsilon \simeq 0.1$.

The thermal stresses which arise in a cylindrical active element are compressional in the interior layers and tensile in the outer layers. The most dangerous stress is set up at the surface of the cylinder, where destruction would be expected to occur first. Prolonged operation – even steady-state operation – is permissible at a high pulse frequency if the thermomechanical stresses do not exceed the limiting permissible stresses for glass. Using $\alpha' = 10.5 \cdot 10^{-6}$ deg^{-1}, $E = 0.75 \cdot 10^6$ kg/cm, and $\nu = 0.24$ for barytic crown glass and a limiting stress of 500 kg/cm^2 [11] to prevent glass destruction, we find the limiting temperature drop of this glass to be 72.4°C. Wide variations in this value are possible under practical conditions because of glass defects and the quality of the glass surface [10, 11]. The limiting temperature drop was checked experimentally by means of thermal shock; in these experiments, glass rods were heated in an oven to a certain temperature and then rapidly transferred to water. The average limiting temperature drop for a large number of rods of various diameters was found to be 72.5°C.

From the nature of the temperature distribution and analysis of the thermal-stress equations, we find that the laser frequency in glass can be raised in several ways. Glasses of different compositions could be used; here a fundamental increase in the limiting temperature drop is possible, primarily because of the coefficient of linear expansion. The surface layer of the active element could be strengthened in various ways [11-13]. Rapid heat exchange could be provided in the center of the rod, in particular through the use of hollow and grooved cylinders as active elements.

NOTATION

T_c	is the coolant temperature;
q	is the heat evolution per unit volume;
c	is the heat capacity per unit volume;
γ	is the density;
λ	is the thermal conductivity;
a	is the thermal conductivity of the active substance;
α	is the heat-exchange coefficient;
$Bi = \alpha R/\lambda$	is the Biot number;
$Fo = at/R^2$	is the Fourier number;
μ_n	are the roots of the equation $J_0(\mu_n)/J_1(\mu_n) = \mu_n/Bi$;
J_0, J_1	are the Bessel functions of the first kind of zeroth and first orders, respectively;
t_0	is the time between pulses;
t	is the instantaneous time during the cooling;
α'	is the coefficient of linear expansion;
E	is the Young's modulus;
ν	is the coolant ratio;
ε	is the ratio of the limiting tensile and compressional stresses.

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